

A Glance after a Century: How the Subject of Cubes were Taught in Geometry Classes at Secondary Schools

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Abstract

This descriptive study was undertaken with the survey model and aimed to present the textual content of geometry classes in 1919 including the examples of application. Document review method was used in the history of education study. The text body under investigation in teaching the subject of cubes in geometry classes was presented in the following format: preparatory stage, reviewing prior lessons, presenting new concepts, associating these new concepts with daily life and student practices. This study which focused on the teaching practices during the historical development process of geometry instruction is believed to contribute to the work of educators in this domain.

Key Words: History of teaching mathematics, secondary school, teaching geometry, cube

Introduction

History of science is defined as the body of research that investigates the development process of scientific knowledge. In this context, it examines the development process of science in different dimensions such as the stages which knowledge has undergone until the present day, the birth and development of scientific theories, dimension of contributions made to scientific knowledge by societies and the methods, tools and devices used by scientists (Topdemir & Unat, 2012).

Stages that the history of science has undergone consist of a rather lengthy process from the first human being on earth to the present time. The efforts of human beings to understand nature and the needs of human beings played a determinant role in the birth of scientific knowledge. Based on these efforts, sciences such as mathematics and geometry were born and an extensive accumulation of knowledge - that has reached our time - has been generated (Topdemir & Unat, 2012)

Education, carried out between adults and youngsters and children and between the ones who know and the ones who do not, is a process that continues since the existence of the first human on earth (Ergün, 2011). Many definitions are provided for education by educators. Ertürk defines education as "the process of creating a desired change in individual's behavior intentionally and via experiences (Ertürk, 1993).

Education and science are two fundamental concepts that simultaneously develop and affect one another. Teaching activities can be considered as the most important fact that combines these two concepts. Education and training play the most important

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role in the transfer of scientific knowledge and in the continuation of knowledge accumulation. In this context, in addition to clarifying the development and change in educational activities from past to present, history of education studies embody the relationship between history of science and education. Presentation of the development and change in education and training activities in all aspects is crucial to establish basis scientific studies. In this study, presenting the past practices of geometry classes based on the studied text will not only contribute to today's geometry teaching practices but it will also create opportunities to make observations and present findings in regards to history of education. In this context, reviewing the stages that geometry, as a course, has undergone in the History of Turkish Education was required.

Examination of the History of Turkish Education shows that both scientific studies related to geometry as a branch of science and teaching geometry as a course in educational institutions date back to Karahanli State, the first Muslim Turkish state. Geometry was always considered to be important in Turkish-Islamic culture and integrated in madrasa curriculums, especially in the Middle Age when great scientific developments were experienced (Akyüz, 2015).

During the Seljuk period, the madrasas, the basic institutions for education and training activities, were the most important science centers of the era. In particular, the Nizamiye Madrasas constituted the most important educational institutions during the Seljuk period. It is known that positive sciences such as Medicine, Surgery, Mathematics, Algebra, Geometry, Astrology and Biology were taught in these madrasas in addition to courses such as Religion, Law, Linguistics and Literature and Philosophy (Akyüz, 2015).

After the first madrasa founded by Orhan Bey in İznik during the foundation of the Ottoman Empire, madrasas reached their peak in terms of education and training activities during the rise of the Ottoman Empire. The Sahn-ı Seman madrasas, founded during the reign of Mehmed the Conqueror became the most important science centers of the world at the time. In the brightest era for the state, Süleymaniye Madrasaları emerged as the highest level education institutions which taught positive sciences as well as religious sciences (Ulusoy, 2007). In the syllabus of these madrasas, geometry titled *hendese* was taught as a separate course from mathematics (Çınar, 2005).

Degeneration started in the education system as well when the fall of the Ottoman Empire started and the positive sciences taught in the madrasas were abandoned. The Ottoman Empire, which started to lose its superiority against the West and lost land with successive defeats since the 18th century, tried to make several innovations in various field in the 19th century. The priority during these initiatives, which were also observed in educational institutions, was given to military education institutions. Military schools founded to train military officers were named as Mühendishane (Engineering Schools) since Mathematics courses were taught under the name Geometry. It can be seen that sciences of mathematics and geometry were started to be taught more in depth in Mühendishane-i Bahr-i Hümayun, opened during the reign of Abdülhamit I. The 4-volume *Mecmua-i Ulum-i Riyaziye* prepared by İshak Efendi was taught as the textbook in Mühendishane-i Berr-i Hümayun, opened during the reign of Selim III. The first volume of the book consisted of algebraic equations and geometry; the second volume included the subjects of plane geometry, analytic geometry and conics along with the subjects of differentials and integral calculus. Establishment of Ottoman public educational institutions whose foundations were laid during the reign of Sultan Mahmud II took a new form with the Reform Period, Statute on General Education of 1869 (Education Act) and reign of Abdülhamid II. Organized education excluding madrasas was graded as primary, secondary and higher education. The geometry course, investigated in the framework of this study, was included in the curriculums of all these learning levels (Demirkan, 2015; Akyüz, 2015; Türk, 2014).

In mathematics teaching, children's intuitive learning, especially in pre-school and primary school, is known to provide an important foundation for future learning (Orbay & Develi, 2015). The significance of this type of learning was also observed during

the Constitutional period when various attempts were made to adapt modern methods of the West to educational activities and intellectuals and educators of the period were involved in important initiatives in this regard. The articles included in the *Tedrisat* (Education) Journal published in Istanbul in 1914 consisted numerous writings on teaching and learning activities, course contents and teaching methods. The article penned by *İsmail Hakkı Baltacıoğlu*, one of the significant educators of the time, titled "*How to Teach Geometry*" published in the *Tedrisat* Journal pointed out the goals of geometry, the place of geometry in daily life and the benefits of geometry for individuals. In addition to these, rather important explanations were also provided about the objectives of geometry and how teaching methods should be established in elementary schools (Oruç & Kırpık, 2006).

The Ottoman educational institutions, which underwent changes and renewals since the 19th century, established the foundations of the education system in the New Turkish State. During the Republican period, significant steps were taken in the existing institutions and with the help of reforms; both schools were restructured and the curriculums were renewed according to modern methods (Aslan, 2011). When this period is examined, it is understood that modernization in the field of education covered all levels of education from primary school to university. It is also seen mathematics and geometry instruction were affected by this change and important steps were taken to reach the level of modern science (Demirkan, 2015).

Geometry maintained its place in these new curriculums as a separate course from mathematics and continued to be taught. It is understood that there were geometry lessons in various classes for various hours at the levels that correspond to today's primary and secondary education levels. The curriculums, prepared in line with the new educational objectives of the Republic and with a scientific approach, were not only created for mathematics and geometry but for all courses as well. Undoubtedly, this transformation gave its first signs in the period before the declaration of the Republic, i.e. during the last periods of the Ottoman Empire and the foundations of the modern scientific approach were started to be experienced by taking Europe as a model (Aslan & Olkun, 2011).

This study examined a sample lesson published in *Tedrisat* Journal in 1919 with the original name «Hendese Dersi Numunesi 'devre-i aliyyede' (mika'blar hakkında)» that can be translated as "An Example of a Geometry Lesson at Primary Schools (On Cubes)". The text includes remarkable points such as the goals of the geometry course during the last period of the Ottoman reign, methods and techniques used in the class and teacher-student dialogues during teaching. The data obtained in this study will contribute to the history of education and mathematics research. In addition, the data presented here will guide the theoretical work of academicians and practical work of teachers in teaching geometry.

This study aimed to investigate how geometry was taught and targets and expected results through a sample applied in the past. The fact that the sample text examined in this study included a lesson plan that was actually implemented and published in the educational journals of the time is significant in understanding the educational approach of the period under question. Moreover, in terms of teaching methods and techniques, the fact that the lesson plan on cubes started from the square to arrive at the concept of cubes by finding the area of the square and by providing equal height and length for the square and that the lesson plan included the calculations for the surface area and volume for the cube are noteworthy.

Method

1. Research model

This study is a descriptive survey study that aimed to present the text content of a sample lesson published in Tedrisat Journal in 1919 with the original name «Hendese Dersi Numunesi 'devre-i aliyede' (mika'blar hakkında)» that can be translated as "An Example of a Geometry Lesson at Primary Schools (On Cubes)". Data collection was undertaken by following the steps of document review method. Document review is a research method that includes the analysis of written materials that contain information on the targeted concept or concepts (Forster, 1995; Patton, 1990).

1.1. Data Collection and Analysis

The sample text investigated in this study was published in the Tedrisat Journal in 1919. The Tedrisat Journal was one of the most prestigious educational journals of the time and its editorial staff primarily included educators (Demir, 2017). The journal consisted of educational articles in a wide range in addition to sample lesson plans.

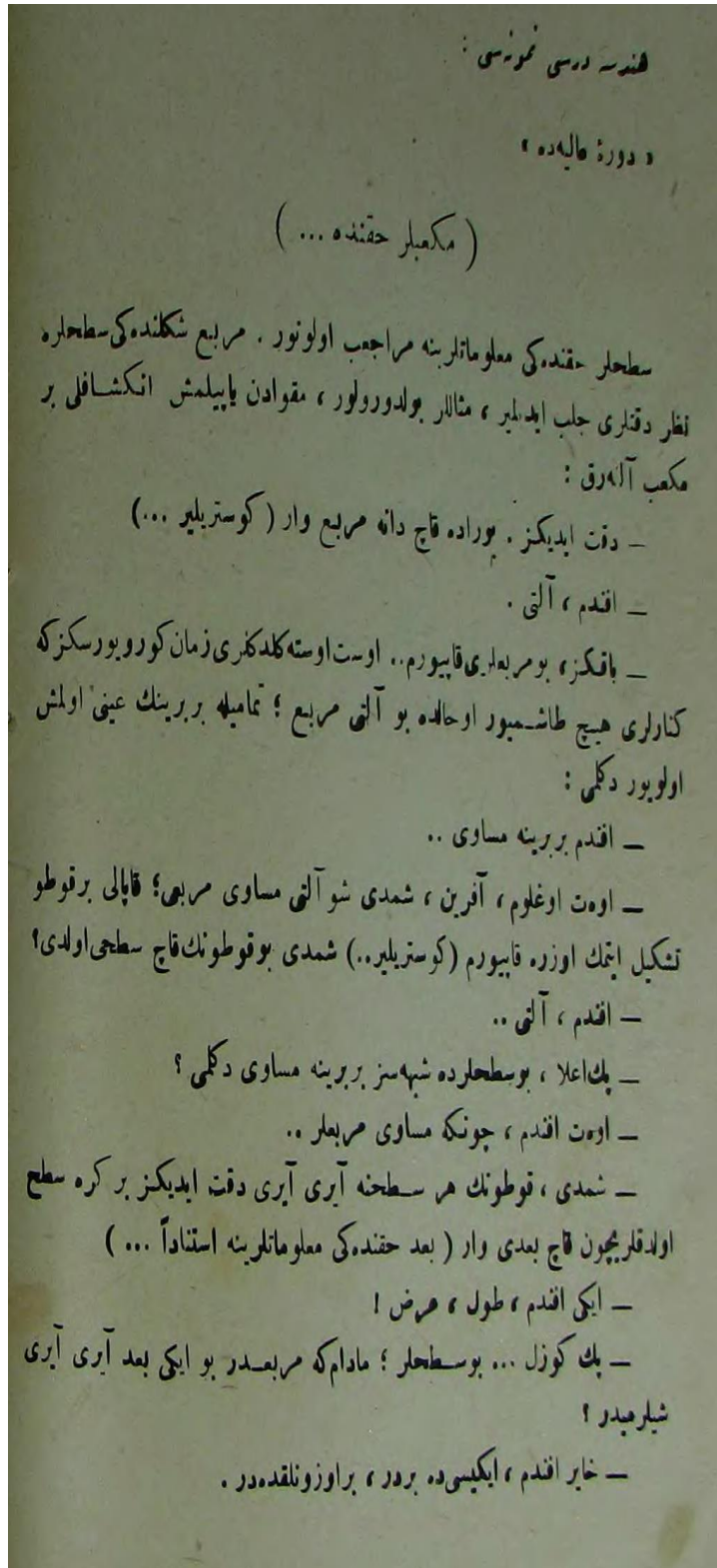
Since the text analyzed in the study was written in Ottoman Turkish, the first step of the study required transcription. Citations from the text simplified by using modern expressions were given in findings section.

2. Findings

The investigated sample geometry lesson published in Tedrisat Journal in 1919 was written by Abdüllatif with the original name «Hendese Dersi Numunesi 'devre-i aliyede' (mika'blar hakkında)» that can be translated as "An Example of a Geometry Lesson at Primary Schools (On Cubes)". The text was presented in the following format;

- Sample lesson plan
- Preparatory phase for the lesson
- Review of prior subjects
- Presentation of the new concepts related to the subject
- Associations between these new concepts and daily life
- Student practices

First of all, the teacher utilized a series of Q&A in order to remind students about their prior learning, to increase their readiness and to motivate them. The text explains the preparatory stage as follows:



Sample Geometry Lesson:

"On Cubes"

Students are reminded about their prior information on surfaces. Their attention is drawn to surfaces in the shape of squares, they are told to locate sample squares. A box made of carton is held and displayed by asking:

-Pay attention to the box, how many squares are there in this box? (pointing)

-Six, sir.

-See, I am folding the squares. When I fold the flaps, they close on one another completely. Therefore, the six squares that you see here are completely the same, aren't they?

-Yes sir, they are equal.

-Yes, that's my boy, well done. Now, I am folding these six equal squares to make a closed box. How many surfaces/sides does this box have now?

-Six, sir

-Well, these surfaces/sides are undoubtedly equal to one another, aren't they?

- Yes, sir, because they are equal squares

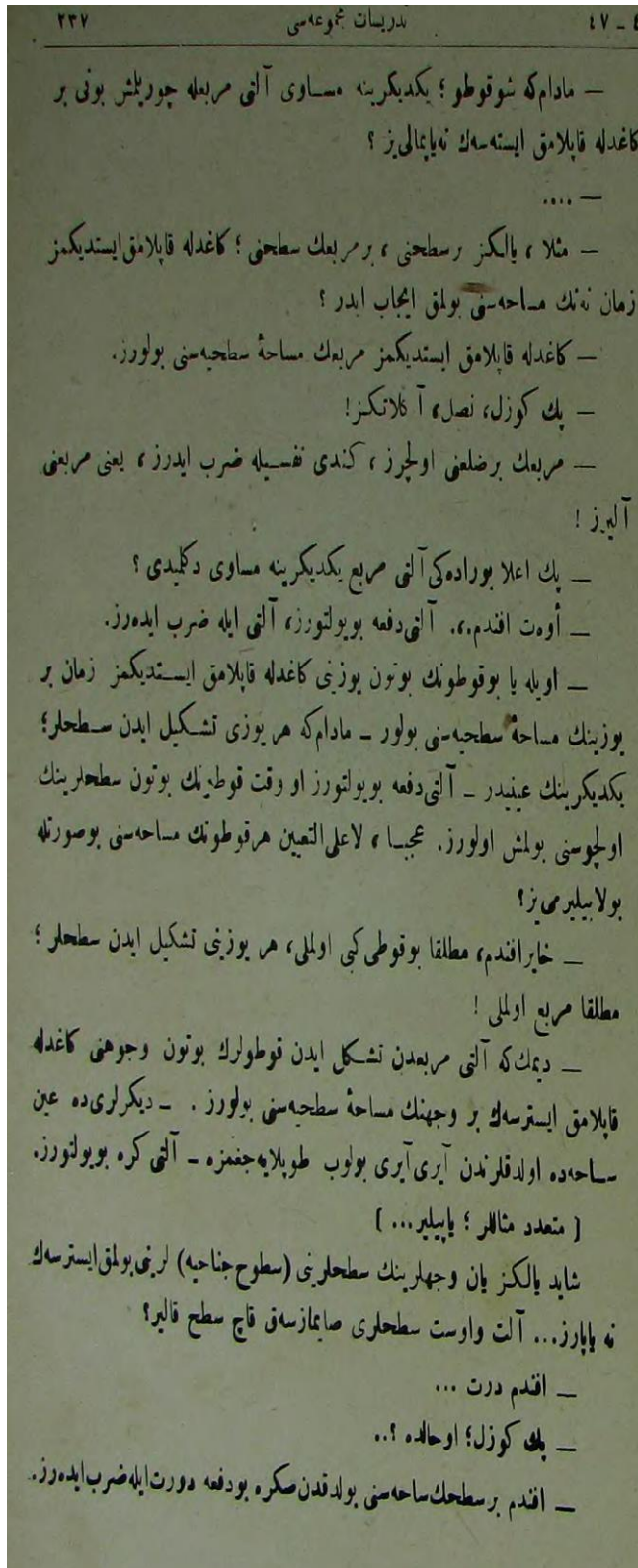
- Now, pay attention to each surface of the box separately, first of all, since they are surfaces, how many measurements are there? (based on their knowledge on measurements)

- Two sir; length and width

- Very well... Since these surfaces are squares, are these two measurements different things?

- No sir, they are both the same. Same length

Figure 1: First page of the original text that was analyzed



Since this box... this box is surrounded by six equal squares; what would you do to cover it with paper?

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- For instance, when we want to cover only one surface, one surface of the square with paper, do we need to find a single surface area?

- We find the surface area for the square we want to cover with paper.

- Very well, how, please explain.

- We measure one length of the square, we multiply it with itself. We square it!

- Very well, aren't all these six squares are equal here?

- Yes sir, we make it bigger six time, we multiply it with six.

- Indeed, when we want to cover the whole surface of this box with paper, we find the surface area of one side- and since the surfaces on each side are equal to others, we amplify it six times. Hence, we find the measurement for all surfaces of the box. I wonder, can we find the surface area for any random box in this way?

- No sir, it has to be like this box, surfaces on each side have to be squares!

- Therefore, if we want to cover all the boxes with six squares with paper, we find the surface are of one part. Since the others also have the same surface areas, instead of adding them one by one, we amplify it by six (provides several examples).

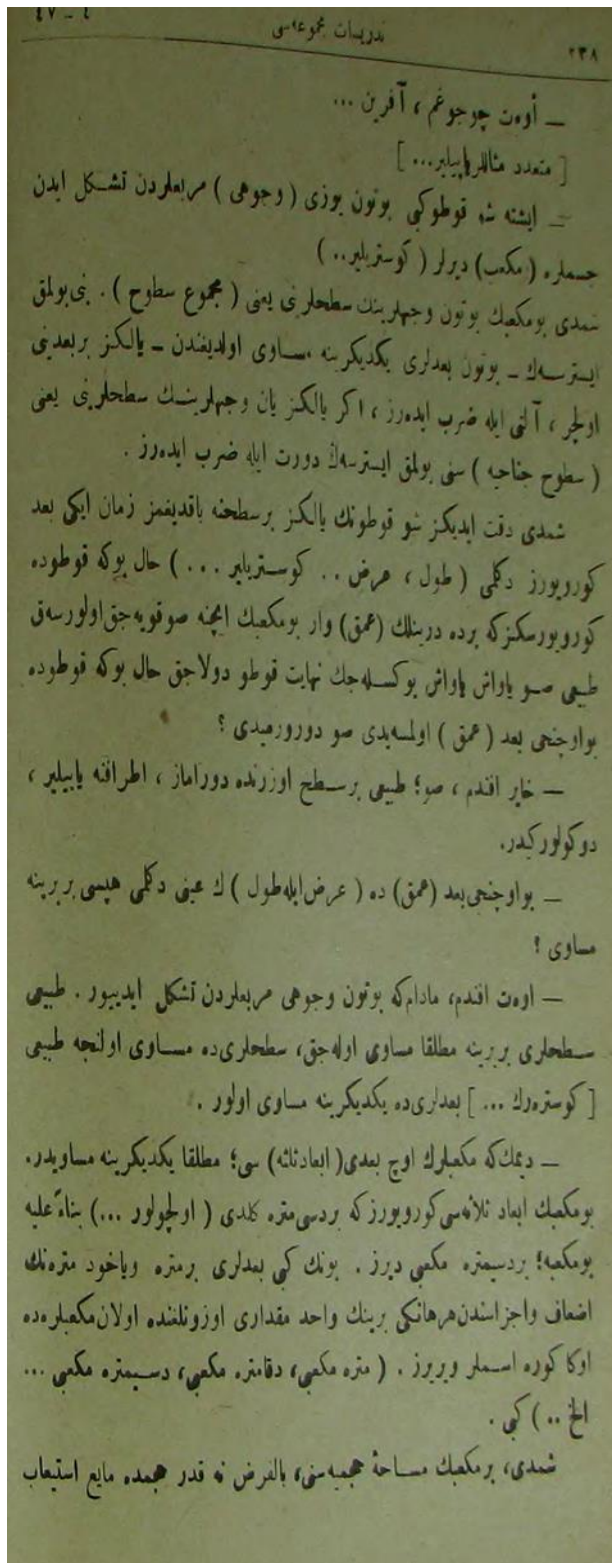
- However, if we want to find the surface area for the lateral sides, what do we do? ... If we do not count the top and bottom sides how many sides are left?

- Sir, four.

- Very well, then?

- Sir, once we find the area of the side, we multiply it by four.

Figure 2: Second page of the original text that was analyzed



Yes, child, well done.

(Several examples are done)

- Objects whose face is composed of squares like this one are called cubes.

Now, if we want to find the sides of all surfaces of this cube (i.e. total area)...since all measurements are equal... we measure only one side and multiply it by six. If we want to find out only the lateral sides, we multiply it by four.

Now, pay attention, when we look at only one surface on this box, we see two measurements right? (length/width, points to both). But, as you see in the box, there is also depth. If we are to put water in this cube, the water will slowly rise and in the end the box will be full. If there was no depth in the box, would the water hold?

- No, sir. Water would not hold on a natural surface; it would overflow and spill.

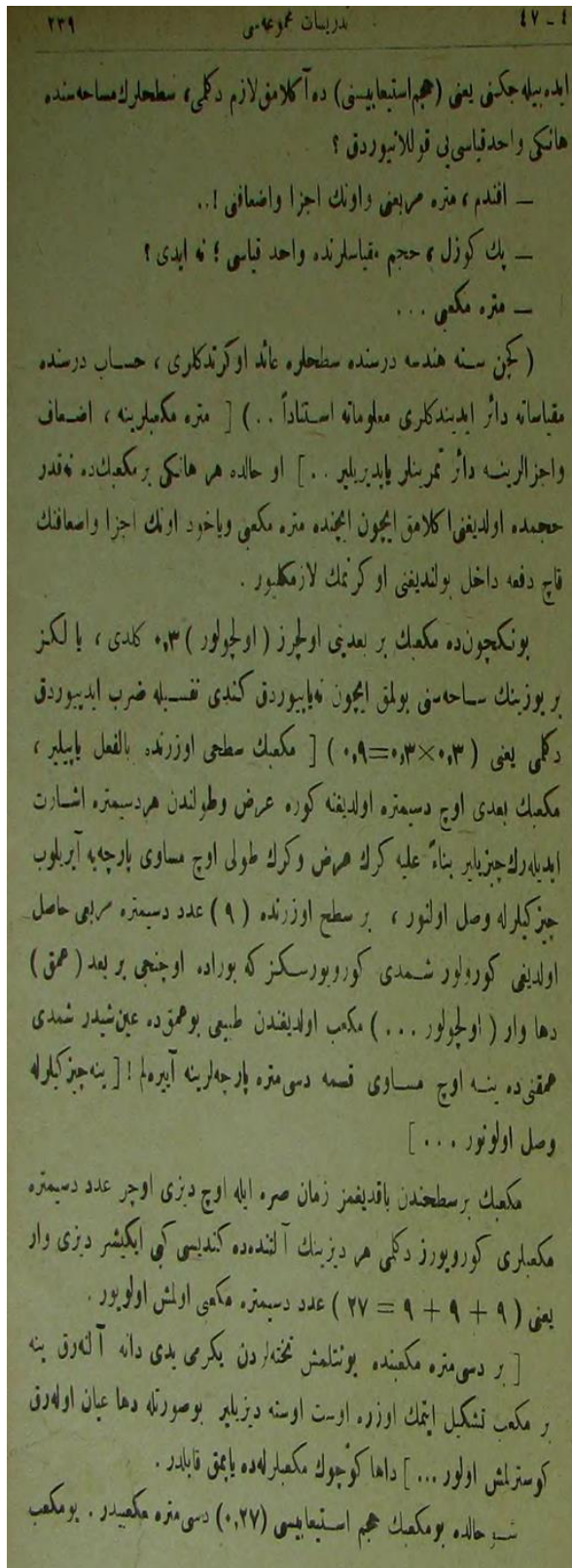
- Isn't this third measurement, depth, the same with length and width? They are all equal.

- Yes, sir; because all the surfaces are made up of squares. Of course, the sides have to be equal. When the sides are equal (shows) the length will also be same.

-Hence, the three measurements for the cubes are always equal to one another. You see the three measurements of this cube, it is one decimeter (measures the cube...). Therefore, we call this cube one decimeter cube. Like this, the cubes whose sides measure a meter or in similar units using a meter are accordingly named (cubic meter, cubic decimeter, cubic centimeter..., etc.).

- Now, it is important to understand the total volume and the volume of liquid it can hold.

Figure 3: Third page of the original text that was analyzed



Right? What unit were we using for surface areas?

- Sir, square meter and its multipliers

- Very well. What unit were we using for measuring volume?

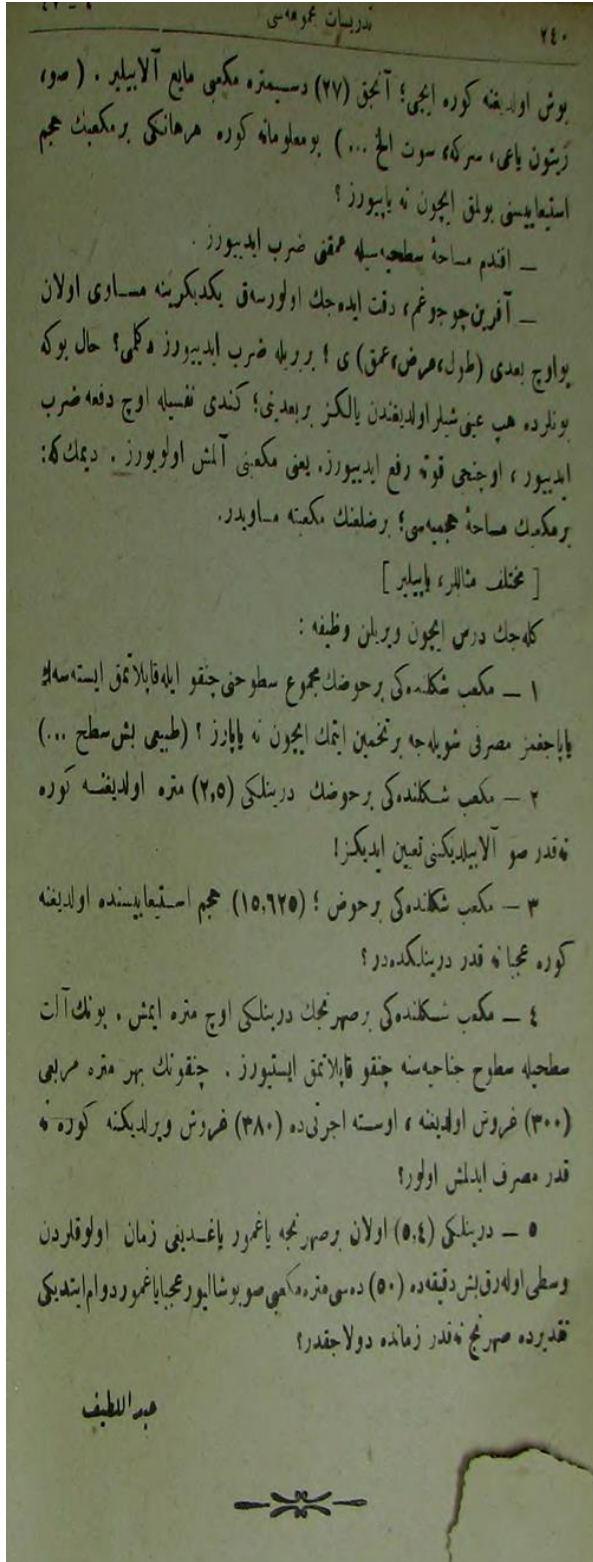
- Cubic meter

(Examples are done on cubic meter multiples and submultiples based on knowledge acquired the previous year in geometry classes about surface measurements).

-In this case; it is necessary to learn the cubic meter of any cube or its multiples and submultiples to measure its volume. In order to do that, we measure the side of the cube (measures it). It is three. What were we doing to find the area of only one side? We multiplied it by itself, don't we? Therefore it is $(3,0 \times 3,0 = 9,0)$ (does the calculation on the surface of the cube, since the length of the cube is three decimeter; points to and draws three decimeters from the length and width. Both length and width are divided into three equal parts and shown by lines. It is seen that there are 9 decimeter squares on each surface). Now you see that there is another measurement here (measures it); since it is a cube it the same with depth. Now, let's divide the depth into three equal parts, equal decimeters (shows the procedure by drawing lines again)

When we look at one face of the cube, we see series of 3 cubes of 3 decimeters don't we? Below each series, there are two series like them therefore it makes $(9+9+9=27)$ decimeters (27 carved wood pieces of one decimeter are stacked on top of another to create a cube. This way, the concept is taught more clearly...). It is possible to make smaller cubes. In this case, total volume of this cube is $(27, 0)$ cubic decimeter.

Figure 4: Fourth page of the original text that was analyzed.



Since this cube is empty, it can only hold 27 cubic decimeter liquid (water, oil, vinegar, milk... etc.). Based on this information, what do we do to find the total volume of any cube?

-Sir, we multiply its surface area of its base by its depth.

-Very well, child; pay attention that we multiply these three equal measurements (length, width and depth). Since they are all the same, we multiply only one side by itself, we take its third power. Hence, we cube it. Therefore, the volume of a cube is equal to the cube of one measurement.

Homework for next lesson:

1. If we are to cover the whole surface of a cube shaped pool with zinc, what do we need to do to estimate the cost? (of course there are five surfaces)
2. Since the depth of a cube shaped pool is (2.5) meter, calculate the amount of water it can hold?
3. Since a cube shaped pool has a total volume of (10,625), how deep it is?
4. The depth of a cube shaped cistern is 3 meters. We want to cover the bottom surface area with zinc. Since each square meter of zinc is (300) cents and craftsmanship costs (380) cents, what is the total cost?
5. When it rains, every five minutes, approximately (50) cubic decimeter water is emptied from the gutters to a cistern with a depth of (5, 4). If the rain continues, how long does it take for the cistern to be filled?

Ali Haydar

Figure 5: Fifth page of the original text that was analyzed

3. Discussion and Result

It is possible to compare and contrast the approaches used in mathematics instruction at that time and today by examining the analyzed lesson plan in terms of teaching mathematics. This lesson plan is significant to compare and contrast the mathematics instruction of the past and today. Teaching geometry at primary school is more important compared to teaching geometry at other grade levels since students at primary schools have their first critical geometry observations, develop intuitions, and acquire concepts and knowledge (Develi & Orbay, 2003). Since students learn about concepts such as triangle and square during this period; geometry instruction during this period lays the foundation of teaching more comprehensive concepts (Sayın & Orbay, 2015). It is important not to disregard the inherent difficulty in some concepts while teaching them. Many concepts in geometry also have inherent difficulties. The child, who is surrounded by many different geometrical objects and concepts, and uses, observes and even plays with daily basic geometrical models in daily life, finds geometrical concepts abstract and complex (Maric & Stamatovic, 2017). However, the lesson plan from a century ago skilfully made it concrete and comprehensible for children to identify cubes, find their surface areas, measure their volumes and solve relevant questions.

In today's education; first of all; student readiness is taken into consideration when a new subject is to be introduced. Students' prior knowledge is tested by starting with the most basic concepts (Sönmez, 2014; Köksal & Atalay, 2016). It was observed that the analyzed text also utilized this approach and attention was paid to ensure that the basic information that was provided could be associated with daily life. The teacher first drew attention to the concept of square, the basis of the cube, and had students find examples. By having them find examples from their surroundings, students were given the sense that mathematics was included in real life and they were also reminded about the characteristics of the square. The teacher had shown students six equal squares made from carton and created a closed box, an object by using these squares. Then, the teacher reinforced the information and motivated the students by explaining that this object had six surfaces, that these surfaces were equal since they were made of equal squares. Later, the teacher drew students' attention to each of these surfaces, i.e. on the square. Using the example of covering the box with paper, the teacher had students accurately express the surface area of the square. By focusing on the object, the teacher had the students find the surface area based on the information that each face of the box were made of equal squares. He even went on to formulate the procedure by having the students calculate the surface area of a side and telling them to multiply it by six. He mentioned that this formula was only relevant for cubes and not for all boxes. He had students come out with the explanation that surface area of one side needed to be multiplied by four in order to find only the lateral surface area. The teacher gave the definition of the cube as well by defining it as the object where all sides were made of equal squares. During the lesson, the teacher summarized what was taught and stated that since all dimensions of a cube was equal, surface area of one face was found by measuring one dimension and squaring it and surface area was found by multiplying it by six and lateral surface area was found by multiplying it by four. This way, the teacher ensured that learning was permanent and what was taught was reinforced. Using repetition intensively or intermittently during learning in order to ensure retention and transfer of what is learned is also suggested by educators today (Köksal & Atalay, 2016; Lancelot, 1944).

In order to proceed to teaching about the volume of cubes, among the concepts of internal, external and rendered volumes; the teacher first used the internal volume concept which is also widely used today. He had the students realize that water could not hold over a surface and that a third dimension was needed for the water to hold. Based on student responses to his questions, the teacher had students comprehend that the third dimension, i.e. the height, was equal to length and width. By holding a cube with a one decimeter length, he stated that this object was called a one cubic decimeter cube and had students understand that the unit length of the cube identified its name (cubic meter, cubic decimeter, cubic centimeter). The teacher went on to remind

students their prior knowledge on units and had them practice. In their study, Raghavan, Sartoris and Glaser (1998) found that students do not fully comprehend the concept of unit and its function in measurement. The fact that the teacher focused on the concept of unit again in the analyzed text also supports the finding that student have difficulty in understanding the concept of unit. In this way, the teacher ensured that students had the required prior knowledge to comprehend the unit used in cubes. He stated that in order to find the volume of any cube, it was necessary to find the number in the cubic meter or its parts or multiples. In order to make this concept more concrete, the teacher used an object which he held in his hand and had students make calculations using this as an example. First of all, the side of a 3-decimeter cube was measured by interacting with students via questions and answers, the teacher made lines on the surface of the cube with 1-decimeter intervals for length and width and generated 9 square parts of 1-decimeter. Including the third dimension, height, the teacher also made lines on the cube with 1-decimeter intervals to create 1-decimeter square parts. Stating that when one looked at the cube from one side, one saw three series of 3-decimeter cubes in a row and each of these series had two similar series like them and showed that there were $9+9+9=27$ unit cubic decimeters. The teaching method expressed as hands on learning, active learning (Sönmez, 2014) or on-the-job training (Küçükahmet, 1998) was identified in this sample text and students were observed to take individual responsibility. It was found that even a century ago, hands on learning was considered to be important and 27 carved wood pieces with cubic meter units were used to create cubes. Also, if we are to pay attention to the use of units again, we realize that while measurements both in text books and in supplementary books are provided in centimeters (the submultiples of meter) today; the analyzed text provided the unit used in cubes as decimeter.

It is seen in the analyzed text that the subject which was taught was continuously recycled. Hence, it was possible for students to fully comprehend the information presented to them and they were allowed to arrive at results. On the other hand, the teacher reminded that *"the measurements of the cube were equal when a student expressed that base area and height should be multiplied when asked about the volume, i.e. the amount of liquid the cube could hold"* and explained that the side should be multiplied by itself three times. Therefore, the teacher provided a type of formula for the solution. When provided with formulas directly, student has difficulty in making sense of the information (Batista & Clements, 1996). The conceptual approach in developing formulas helps students learn how to effectively use these tools to measure the different characteristics of objects around them (Van de Walle, Karp & Bay Williams, 2013). In the analyzed text, rather than providing the formula to students to practice, the teacher had students come up with the formula and did not let students have difficulty in making sense of what they learned. Here we observe the discovery learning technique because in this technique, students are not given the results at the beginning of the lesson. Based on a problem that will pique student interest, students are guided to come up with the result on their own by using questions from simple to complex, from concrete to abstract and from near to far. It was seen in the analyzed text that this method suggested by Bruner in 1960's (Bruner, 1962) was skillfully used in 1919.

Homework is still a controversial issue among educators. While some educators oppose homework, some believe that homework is a crucial task to reinforce learning. Homework is regarded to be important to develop study skills and to repeat what has been learned (Küçükahmet, 1998; Hesapçioğlu, 2011; Bempechat, 2014; Zimmerman & Kitsantas, 2005). The analyzed text also included homework at the end of the lesson. The homework was to be done until the next class. Examination of the homework given in the sample lesson plan shows that it included daily life examples; it proceeded from easy to hard and covered other aspects of mathematics.

Another noteworthy aspect in the analyzed text is related to courtesy and respect in teacher-student interactions. It is clear that the teacher provided a democratic climate in which students could freely express their thoughts and information could be revealed easily. The use of the word "Sir" in student responses reflects the respect felt by students to their teacher.

When the text, which was provided in the Findings Section in detail, was evaluated; it is possible to make inferences not only about the lesson content and type of instruction but also about the instructional approach used at the time. It was observed in the analyzed text that education was based on student-centered approach, discovery learning technique was utilized and students were especially active in using lesson materials and tools. A teaching style which proceeded from simple to complex was adopted, student readiness levels were identified especially while introducing the new subject and they were motivated by using their prior knowledge. After the new information and rules were given, students were allowed to practice and additional homework was given to reinforce what was taught.

Suggestions

This study examined the text whose title was translated as "An Example of a Geometry Lesson at Primary Schools (On Cubes)" which was published in the *Tedrisat Journal* in 1919 and compared it with modern educational approaches. It is rather striking to observe that the educational philosophies that are valid today were being used at that time when they were not even identified yet. In this context, light will be shed on the history of educational philosophies that are common today by translating and interpreting educational documents used in the past.

Teaching of some concepts is rather hard due to inherent difficulties that exist in the nature of these concepts. Volume is also one of these concepts which is difficult to learn and which involves misconceptions. Examining how this concept was taught in the past will provide a new outlook to teachers in teaching the subject of volume. Examining prior work undertaken both in mathematics and in other sciences will help review the teaching of many concepts which students find difficult.

References

- Akyüz, Y. (2015). *Türk eğitim tarihi*. Ankara: Pegem Akademi.
- Aslan E. & Olkun S. (2011). Elementary school mathematics in the first curricula of Turkish Republic. *Elementary Education Online*, 10(3), 991-1009, <http://ilkogretim-online.org.tr>
- Aslan, E. (2011). Türkiye Cumhuriyeti'nin ilkokullarda izlediği ilköğretim programı: "1924 İlk Mektepler müfredat programı". *İlköğretim Online*, 10 (2), 717-734.
- Batista, M.T. & Clements, D. H. (1996) Students' understanding of three- dimensional rectangular arrays of cubes. *Journal of Research in Mathematics*, 27, 258-292.
- Bempechat J. (2004) The motivational benefits of homework: A social cognitive perspective. *Theory Into Practice*, 43 (3), 189-196.
- Bruner, J. S. (1962). *On Knowing Essays For the Left Hand*. Cambridge: Harvard University Press.
- Çınar S. (2005). *Eskişehir eğitim tarihi (1876–2004)*. (Yüksek Lisans Tezi). Eskişehir Osmangazi Üniversitesi Sosyal Bilimler Enstitüsü, Eskişehir.
- Demir, G.T. (2017). Atatürk döneminde yayınlanan eğitim dergilerinde sosyal bilgiler konularının incelenmesi (1923-1938). Ankara: Atatürk Araştırma Merkezi.

- Demirkan, A. (2015). Atatürk döneminde matematik eğitimi (Yüksek lisans tezi). Dokuz Eylül Üniversitesi Atatürk İlke ve İnkılap Tarihi Enstitüsü, İzmir.
- Develi, M.H. & Orbay, K. (2003). İlköğretimdeniçin ve nasıl birgeometri öğretimi. *Milli Eğitim Dergisi*, 157, 115-122.
- Ergün, M. (2011). Eğitim felsefesi. Ankara: Pegem Akademi.
- Ertürk, S. (1993). Eğitimde program geliştirmeye. Ankara: Hacettepe Üniversitesi Basımevi.
- Forster, N. (1995). The analysis of company documentation. In Cassell, C. & Symon, G. (Eds.), *Qualitative Methods in Organizational Research: A Practical Guide*. London: Sage.
- Hesapçıoğlu, M. (2011). Öğretim ilke ve yöntemleri. Ankara: Nobel Yayıncılık.
- Köksal, O. & Atalay, B. (2016). Öğretim ilke ve yöntemleri. Konya: Eğitim Kitabevi.
- Küçükahmet, L. (1998). Öğretim ilke ve yöntemleri. İstanbul: Alkım Yayınları.
- Lancelot, W.H. (1944). Permanent Learning: A Study in Educational Techniques. Oxford, England: Wiley; Chapman & Hall, Ltd.
- Marivic, S.M. & Stamatovic, J.D. (2017). The effect of preschool mathematics education in development of geometry concepts in children. *EURASIA Journal of Mathematics Science and Technology Education*, 13 (9), 6175-6187.
- Orbay, K. & Develi, M.H. (2015). Pre-school students' informal acquisitions regarding the concepts of point and straight line. *International Electronic Journal of Elemenary Education*, 7(3), 319-332.
- Oruç, Ş. & Kırpık, G. (2006). Osmanlı'da modern öğretim, strateji, yöntem ve teknikleri (Tedrisat Mecmuasından makaleler). Ankara: Gazi Kitabevi.
- Patton, M. (1990). Qualitative Evaluation and Research Methods. (2nd ed.). Newbury Park, CA: Sage.
- Raghavan, K., Sartoris, M.L. & Glaser, R. (1998). Interconnecting science and mathematics concepts. In Lehrer, R. & Chazan, D. (Eds.), *Designing learning environments for developing understanding of geometry and space*. Hillsdale, NJ: LEA Publishers.
- Sayın, V. & Orbay K. (2015). Investigation of 4th grade students' geometric thinking levels and success scores in terms of different variables. *Journal of Studies in Social Sciences*, 12 (1), 160-179.
- Sönmez, V. (2014). Öğretim ilke ve yöntemleri. Ankara: Pegem Akademi.
- Topdemir, H.G. & Unat, Y. (2012). Bilim tarihi. Ankara: Pegem Akademi.
- Türk, İ.C. (2014). "Türkiye'de Orta Tahsil" Başlıklı Risaleye Göre Osmanlı İmparatorluğu'ndan Ulus Devlet Türkiye'ye İntikal Eden Ortaöğretim Mirası. *A. Ü. Türkiyat Araştırmaları Enstitüsü Dergisi [TAED]* 52, 351-378.
- Ulusoy A. (2007). Kuruluşundan 17. yüzyıla kadar Osmanlı medreselerinde eğitim-öğretim faaliyetleri (Yüksek lisans tezi). Selçuk Üniversitesi Sosyal Bilimler Enstitüsü, Konya.
- Van de Walle, J.A., Karp, K.S. & Bay-Williams, J.M. (2013). Elementary and Middle School Mathematics (8th Ed.). Boston: Pearson Education Inc.
- Zimmerman, B.J. & Kitsantas, A. (2005). Homework practices and academic achievement: The mediating role of self-efficacy and perceived responsibility beliefs. *ELSEVIER Contemporary Educational Psychology*, 30, 397-417.